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Electronic Companion

This e-companion provides supplementary material for the manuscript *"Schedule Design for Liner Services under Vessel Speed Reduction Incentive Programs"*.

EC.1. Table of abbreviations

Table EC.1 Table of abbreviations

Abbreviation	Explanation
γ nm VSRZ	A Vessel Speed Reduction Zone with a radius of γ nautical miles
APL	American President Lines
chosen VSRZ	A Vessel Speed Reduction Zone in which its rules are complied with by the studied shipping company
CO_2	Carbon dioxide
GFIP	Green Flag Incentive Program
KKT	Karush-Kuhn-Tucker
knots	Nautical miles per hour
LB	Lower bound
LGB	The Port of Long Beach
LSA	The Port of Los Angeles
nm	Nautical miles
non-VSRIP port	A port without any Vessel Speed Reduction Incentive Program
non-VSRZ leg	A sailing along the shortest path between two consecutive ports of call that are both non-VSRIP ports
non-zero VSRZ	A γ nm VSRZ when γ is greater than 0
NO_x	Nitrogen oxides
NYK	The Port of New York
out-VSRZ leg	A sailing along the shortest path outside a chosen Vessel Speed Reduction Zone that is between two consecutive ports of call with a VSRIP port (A voyage along the shortest path between two consecutive ports of call with a VSRIP port is regarded as two legs, i.e., the sailing within the zone and the sailing outside the zone.)
PM	Particulate matter
SDI	The Port of San Diego
SO_2	Sulfur dioxide
TEU	Twenty-foot equivalent unit
UB	Upper bound
VSRIP	Vessel Speed Reduction Incentive Program
VSRIP port	A port adopting a VSRIP
VSRZ	Vessel Speed Reduction Zone
in-VSRZ leg	A sailing along the shortest path within a chosen VSRZ that is between two consecutive ports of call with a VSRIP port (A voyage along the shortest path between two consecutive ports of call with a VSRIP port is regarded as two legs, i.e., the sailing within the zone and the sailing outside the zone.)
VSRZ plan	A plan includes the total chosen VSRZs of all VSRIP ports

EC.2. Proof of Lemma 1

Proof. We apply the Karush-Kuhn-Tucker (KKT) conditions to analyze the properties of the optimal sailing speed for each leg of route r without considering the speed limit constraints (4). Let μ and ν_l be the Lagrangian multipliers associated with constraints (3) and (5), respectively.

The linearity constraint qualification holds since both constraints (3) and (5) are linear. The KKT conditions are

$$-w\cdot\alpha_{i_r}\cdot\beta_{i_r}\cdot d_{krl}^{\beta_{i_r}+1}\cdot t_{rl}^{-(\beta_{i_r}+1)}+\mu-\nu_l=0,\forall l\in L_r \tag{EC.1}$$

$$\nu_l\cdot t_{rl}=0,\forall l\in L_r \tag{EC.2}$$

$$\nu_l\geq 0,\forall l\in L_r \tag{EC.3}$$

$$\sum_{l\in L_r}t_{rl}=T\cdot q_r-T_r \tag{EC.4}$$

$$t_{rl}\geq 0,\forall l\in L_r, \tag{EC.5}$$

where (EC.1) is the KKT equation, (EC.2) is the complementary slackness condition, (EC.3) ensures Lagrangian multipliers are nonnegative, (EC.4) and (EC.5) impose the feasibility of the solution. Since $t_{rl}>0$ for all $l\in L_r$ in the optimal solution, we infer that $\nu_l=0$ for all $l\in L_r$ by (EC.2). Then, according to (EC.1), the optimal sailing speed of leg l, denoted by $\hat{v}_{krl}$, can be written as

$$\hat{v}_{krl}=\left(\frac{\mu}{w\cdot\alpha_{i_r}\cdot\beta_{i_r}}\right)^{\frac{1}{\beta_{i_r}+1}},\forall l\in L_r. \tag{EC.6}$$

Therefore, if constraints (4) are not considered, the optimal sailing speeds of all legs on the same route are equal in model [P0]. □

EC.3. Proof of Lemma 2

Proof. We prove the lemma by contradiction. When q_r^1 is increased to q_r^2, assume that there exists an optimal solution with a leg $\hat{l}$ whose optimal sailing time will decrease, i.e., $\hat{t}^1_{kr\hat{l}}>\hat{t}^2_{kr\hat{l}}$. Then, define $\hat{\delta}=\hat{t}^1_{kr\hat{l}}-\hat{t}^2_{kr\hat{l}}$ and there are some legs $\tilde{l}_1,\tilde{l}_2,...,\tilde{l}_V$ in the solution whose optimal sailing times are increased with $\tilde{\delta}_1$ ($\tilde{\delta}_1=\hat{t}^2_{kr\tilde{l}_1}-\hat{t}^1_{kr\tilde{l}_1}>0$), $\tilde{\delta}_2,...,\tilde{\delta}_V$, where $\tilde{\delta}_1+\tilde{\delta}_2+...+\tilde{\delta}_V\geq\hat{\delta}$, $1\leq V\leq|L_r|-1$. Take some positive values $\delta_1,\delta_2,...,\delta_V$ such that $\delta_1\leq\tilde{\delta}_1,\delta_2\leq\tilde{\delta}_2,...,\delta_V\leq\tilde{\delta}_V$ and $\delta_1+\delta_2+...+\delta_V=\hat{\delta}$. We present a new function on the fuel cost of leg l:

$$g_{krl}(t_{rl}):=w\cdot d_{krl}\cdot\alpha_{i_r}(\frac{d_{krl}}{t_{rl}})^{\beta_{i_r}}. \tag{EC.7}$$

Since $\hat{t}^1_{kr\hat{l}}$, $\hat{t}^1_{kr\tilde{l}_1},\hat{t}^1_{kr\tilde{l}_2},...,\hat{t}^1_{kr\tilde{l}_V}$ are the optimal sailing times for legs $\hat{l}$, $\tilde{l}_1,\tilde{l}_2,...,\tilde{l}_V$ when q_r^1 ships are deployed, the fuel cost will not decrease if the time reduced by leg $\hat{l}$ is added on legs $\tilde{l}_1,\tilde{l}_2,...,\tilde{l}_V$, i.e., $g_{kr\hat{l}}(\hat{t}^1_{kr\hat{l}}-\hat{\delta})+\sum_{v=1}^V g_{kr\tilde{l}_v}(\hat{t}^1_{kr\tilde{l}_v}+\delta_v)\geq g_{kr\hat{l}}(\hat{t}^1_{kr\hat{l}})+\sum_{v=1}^V g_{kr\tilde{l}_v}(\hat{t}^1_{kr\tilde{l}_v})$. The first and second order derivatives of $g_{krl}(t_{rl})$ are

$$\frac{dg_{krl}(t_{rl})}{dt_{rl}}=-w\cdot\alpha_{i_r}\cdot\beta_{i_r}\cdot {d_{krl}}^{\beta_{i_r}+1}\cdot {t_{rl}}^{-(\beta_{i_r}+1)}<0 \tag{EC.8}$$

$$\frac{d^2g_{krl}(t_{rl})}{d{t_{rl}}^2}=w\cdot\alpha_{i_r}\cdot\beta_{i_r}\cdot(\beta_{i_r}+1)\cdot {d_{krl}}^{\beta_{i_r}+1}\cdot {t_{rl}}^{-(\beta_{i_r}+2)}>0. \tag{EC.9}$$

We can see that the function $g_{krl}(t_{rl})$ is strictly convex in t_{rl}, and the first order derivative of $g_{krl}(t_{rl})$ is negative but increases with t_{rl}. Hence $g_{kr\tilde{l}_v}(\hat{t}^2_{kr\tilde{l}_v}) - g_{kr\tilde{l}_v}(\hat{t}^1_{kr\tilde{l}_v} + \delta_v) > g_{kr\tilde{l}_v}(\hat{t}^2_{kr\tilde{l}_v} - \delta_v) - g_{kr\tilde{l}_v}(\hat{t}^1_{kr\tilde{l}_v})$, $v = 1, 2, ..., V$. As a result, we conclude that $g_{kr\hat{l}}(\hat{t}^2_{kr\hat{l}}) + \sum_{v=1}^{V} g_{kr\tilde{l}_v}(\hat{t}^2_{kr\tilde{l}_v}) = g_{kr\hat{l}}(\hat{t}^1_{kr\hat{l}} - \hat{\delta}) + \sum_{v=1}^{V} g_{kr\tilde{l}_v}(\hat{t}^1_{kr\tilde{l}_v} + \delta_v) + \sum_{v=1}^{V} \left[g_{kr\tilde{l}_v}(\hat{t}^2_{kr\tilde{l}_v}) - g_{kr\tilde{l}_v}(\hat{t}^1_{kr\tilde{l}_v} + \delta_v)\right] \geq g_{kr\hat{l}}(\hat{t}^1_{kr\hat{l}}) + \sum_{v=1}^{V} g_{kr\tilde{l}_v}(\hat{t}^1_{kr\tilde{l}_v}) + \sum_{v=1}^{V} \left[g_{kr\tilde{l}_v}(\hat{t}^2_{kr\tilde{l}_v}) - g_{kr\tilde{l}_v}(\hat{t}^1_{kr\tilde{l}_v} + \delta_v)\right] > g_{kr\hat{l}}(\hat{t}^1_{kr\hat{l}}) + \sum_{v=1}^{V} g_{kr\tilde{l}_v}(\hat{t}^1_{kr\tilde{l}_v}) + \sum_{v=1}^{V} \left[g_{kr\tilde{l}_v}(\hat{t}^2_{kr\tilde{l}_v} - \delta_v) - g_{kr\tilde{l}_v}(\hat{t}^1_{kr\tilde{l}_v})\right] = g_{kr\hat{l}}(\hat{t}^1_{kr\hat{l}}) + \sum_{v=1}^{V} g_{kr\tilde{l}_v}(\hat{t}^2_{kr\tilde{l}_v} - \delta_v)$, where $\hat{t}^2_{kr\hat{l}} + \sum_{v=1}^{V} \hat{t}^2_{kr\tilde{l}_v} = \hat{t}^1_{kr\hat{l}} + \sum_{v=1}^{V} (\hat{t}^2_{kr\tilde{l}_v} - \delta_v)$ and $\hat{t}^1_{kr\tilde{l}_v} \leq \hat{t}^2_{kr\tilde{l}_v} - \delta_v \leq \hat{t}^2_{kr\tilde{l}_v}$, $v = 1, 2, ..., V$, indicating that the new solution with sailing time $\hat{t}^1_{kr\hat{l}}$ for leg $\hat{l} \in L_r$ and sailing times $\hat{t}^2_{kr\tilde{l}_v} - \delta_v$, $v = 1, 2, ..., V$ for legs $\tilde{l}_v \in L_r$, $v = 1, 2, ..., V$ is feasible. The result shows that, when the number of deployed ships is q^2_r, increasing the sailing time of $\hat{\delta}$ on leg $\hat{l}$ by reducing the same total time from legs $\tilde{l}_1, \tilde{l}_2, ..., \tilde{l}_V$ leads to a lower cost. We have therefore proved that, when q_r increases, there does not exist leg $\hat{l}$ whose optimal sailing time will decrease. □

EC.4. Proof of Proposition 1

Proof. L^S_{kr} is designed as an empty set and L^N_{kr} includes all legs in L_r. We determine the legs with the lowest speed limit (denoted by $V^{\min}_r$) of all legs in L^N_{kr}. We set $V^{\min}_r$ as the initial speeds of legs in L^N_{kr}. The number of ships required can be calculated as (EC.10), which may not be an integer number.

$$q'_{kr} = \frac{\sum_{l \in L^N_{kr}} d_{krl}/V^{\min}_r + T_r}{T}. \tag{EC.10}$$

There are three cases obtained by comparing q'_{kr} and q_r:

(i) If $q'_{kr} = q_r$, $V^{\min}_r$ is the optimal speed of all legs by Lemma 1.

(ii) If $q'_{kr} < q_r$, according to the same lemma, the optimal speed of legs in L^N_{kr} is $\sum_{l \in L^N_{kr}} d_{krl}/(T \cdot q_r - T_r)$, which is less than speed limits V_{rl} of all legs $l \in L^N_{kr}$.

(iii) If $q'_{kr} > q_r$, we can reduce q'_{kr} by increasing the speeds of some legs, and speed limits thus should be considered. We redesign the optimal speeds for the legs on route r as follows. First, we shift the legs whose speed limits are equal to $V^{\min}_r$ from L^N_{kr} to L^S_{kr}. According to Lemma 2, the optimal speeds of legs will not decrease when q'_{kr} decreases. Hence, we conclude that the optimal speeds of legs in L^S_{kr} are equal to their speed limits with the decrease of q'_{kr}. We search the legs with the lowest speed limit from L^N_{kr} and assign the lowest speed limit to $V^{\min}_r$. $V^{\min}_r$ will be regarded as the new speed of all legs in L^N_{kr}, demonstrating the speed of these legs is increased. The new q'_{kr} is

$$q'_{kr} = \frac{\sum_{l \in L^N_{kr}} d_{krl}/V^{\min}_r + \sum_{l \in L^S_{kr}} (d_{krl}/V_{rl}) + T_r}{T}. \tag{EC.11}$$

Second, we repeat the first step until q'_{kr} is no bigger than the given q_r, and then shift the legs whose speed limits are equal to $V^{\min}_r$ from L^N_{kr} to L^S_{kr}. Define L^S_{kr} and L^N_{kr} as $\hat{L}^S_{kr}$ and $\hat{L}^N_{kr}$. The optimal speed of all legs in $\hat{L}^N_{kr}$ is $\sum_{l \in \hat{L}^N_{kr}} d_{krl}/[T \cdot q_r - \sum_{l \in \hat{L}^S_{kr}} (d_{krl}/V_{rl}) - T_r]$ by Lemma 1.

Therefore, for a given plan k and a given ship number q_r on route r, the optimal speeds of legs in $\hat{L}_{kr}^S$ are equal to their speed limits, which are lower than the speed limits of legs in $\hat{L}_{kr}^N$, and the optimal speeds of remaining legs are the same and are lower than their speed limits. $\square$

EC.5. Proof of Proposition 2

Proof. We put forward a new function on the fuel cost of route r for plan k:

$$h_{kr}(\boldsymbol{t}_r) := \sum_{l \in L_r} g_{krl}(t_{rl}). \tag{EC.12}$$

The Hessian matrix H of $h_{kr}(\boldsymbol{t}_r)$ is

$$H = \begin{bmatrix} \frac{d^2 g_{kr1}(t_{r1})}{d{t_{r1}}^2} & 0 & \cdots & 0 \\ 0 & \frac{d^2 g_{kr2}(t_{r2})}{d{t_{r2}}^2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{d^2 g_{kr|L_r|}(t_{r|L_r|})}{d{t_{r|L_r|}}^2} \end{bmatrix}. \tag{EC.13}$$

Hence $h_{kr}(\boldsymbol{t}_r)$ is a strictly convex function since all order leading principal minors of the Hessian matrix H are positive. We define another function $\tilde{f}_{kr}(\tilde{q}_r)$:

$$\tilde{f}_{kr}(\tilde{q}_r) := \overline{c}_{i_r} \cdot \tilde{q}_r + \underset{\boldsymbol{t}_r}{\text{minimize}}\, h_{kr}(\boldsymbol{t}_r), \tag{EC.14}$$

where $\tilde{q}_r$ is a real number belonging to $\mathbb{R}$. For a given $\tilde{q}_r' \in [q_r', q_r'+1]$, where $q_r' \in \mathbb{Z}$, $\lambda \in [0,1]$, and $\tilde{q}_r' = \lambda q_r' + (1-\lambda)(q_r'+1)$, we have $\lambda \tilde{f}_{kr}(q_r') + (1-\lambda)\tilde{f}_{kr}(q_r'+1) = \lambda h_{kr}(\hat{\boldsymbol{t}}_{kr}') + (1-\lambda) h_{kr}(\hat{\boldsymbol{t}}_{kr}'') + \lambda \overline{c}_{i_r} q_r' + (1-\lambda)\overline{c}_{i_r}(q_r'+1) \geq h_{kr}(\lambda \hat{\boldsymbol{t}}_{kr}' + (1-\lambda)\hat{\boldsymbol{t}}_{kr}'') + \overline{c}_{i_r}[\lambda q_r' + (1-\lambda)(q_r'+1)] \geq \tilde{f}_{kr}(\lambda q_r' + (1-\lambda)(q_r'+1)) = \tilde{f}_{kr}(\tilde{q}_r')$, where $\hat{t}_{krl}'$ and $\hat{t}_{krl}''$ are the optimal sailing times on leg l of route r in plan k for $\tilde{f}_{kr}(q_r')$ and $\tilde{f}_{kr}(q_r'+1)$, respectively. Therefore, $\tilde{f}_{kr}(\tilde{q}_r')$ is a convex function in each interval of $[q_r', q_r'+1]$, meaning $\tilde{f}_{kr}(\tilde{q}_r)$ can be regarded as the local convex extension of $f_{kr}(q_r)$. Similarly, we can prove that $\tilde{f}_{kr}(\tilde{q}_r)$ is strictly convex on $\mathbb{R}$. Hence, we conclude that the function $f_{kr}(q_r)$ is strictly integrally convex referring to Moriguchi and Murota (2019). $\square$

EC.6. Proof of Proposition 3

Proof. The NP-hardness of our problem is proved by reduction from the 0-1 Knapsack Problem which is a well-known NP-complete problem. The 0-1 Knapsack Problem can be described as follows: Given a weight capacity T', a set of goods $P = \{1, ..., n\}$, a weight t_p and a value c_p for each good $p \in P$, the target of the problem is to determine the optimal choice of goods by binary variables x_p for all $p \in P$ to achieve the maximum value ($\text{maximize} \sum_{p \in P} c_p \cdot x_p$) within the capacity ($\sum_{p \in P} t_p \cdot x_p \leq T'$).

Based on the 0-1 Knapsack Problem, we put forward a simplified instance of our problem designed as follows. There is only one route r in the studied company, where each port of call is a different physical port from the others. Each port $p \in \{1,...,n\}$ on the route adopts a separate VSRIP with one non-zero VSRZ. Each ship visit can gain a dockage refund c_p if the operating company participates in the VSRIP at port p. We define both fuel cost and operating cost equal to zero. Let the maximum physical speed of ships sailing outside VSRZs be infinity and hence the total sailing time outside VSRZs can be regarded as zero. The total time spent at all ports of call on the route is T_r. Then the sailing time allowed for sailing within VSRZs on the route, denoted by T', is $T \cdot Q_{i_r} - T_r$. The sailing distance and the speed limit within the non-zero VSRZ of port p are d_p and V_p^U, $p \in P$, respectively. The minimum sailing time t_p within the zone can therefore be calculated as d_p/V_p^U, $p \in P$. x_p is a binary variable used to determine whether the company joins in the VSRIP at port p for each $p \in P$ or not. In this instance, we observe that: (i) The compliance of VSRIPs at different ports is independent. (ii) A ship visit takes at least t_p to sail within the chosen non-zero VSRZ at port p and can achieve a refund c_p. (iii) The maximum dockage refunds for the route can be computed by maximizing $\sum_{p\in P} c_p \cdot x_p$ with the sailing time limit $\sum_{p\in P} t_p \cdot x_p \le T'$.

It follows easily that the simplified instance can gain the maximum dockage refunds if and only if there exists an optimal solution for the 0-1 Knapsack Problem, indicating the 0-1 Knapsack Problem can be solved by solving the instance. The simplified instance is NP-hard, so is our problem considering fuel cost, operating cost, port time, sailing time outside VSRZs and a large number of routes. □

EC.7. Algorithm for solving model [P2]

We first add two new parameters as follows:

Parameters

θ_{kr,q_r} Increased cost for route $r \in R$ in plan $k \in K$ when the number of deployed ships is reduced from q_r to $q_r - 1$

q^*_{kr} Optimal integer number of deployed ships on route $r \in R$ for plan $k \in K$ to minimize the total cost of model [P2]

We calculate the increased cost for route r in plan k when one ship is reduced from the route with q_r ships.

$$\theta_{kr,q_r} := f_{kr}(q_r - 1) - f_{kr}(q_r), \tag{EC.15}$$

where $q_r \in \{q^{\min}_{kr}+1, q^{\min}_{kr}+2, ..., \hat{q}_{kr}\}$. We have proved that the function $f_{kr}(q_r)$ is strictly integrally convex in Proposition 2. Then we have $\theta_{kr,q_r} > \theta_{kr,q_r+1}$ for all $q_r \in \{q^{\min}_{kr}+1, q^{\min}_{kr}+2, ..., \hat{q}_{kr}-1\}$. Define $\theta_{kr,q^{\min}_{kr}}$ to be infinity on route r for all $r \in R$. Algorithm EC.7.1 (referring to the pseudo-polynomial-time algorithm in Wang (2016)) is proposed for solving model [P2] to gain the optimal solution q^*_{kr} for all $r \in R$.

Algorithm EC.7.1 Compute the optimal solution of plan k

Input $\hat{q}_{kr}$ and θ_{kr,q_r} for all $r \in R$ and $q_r \in \{q_{kr}^{\min}, q_{kr}^{\min}+1, ..., \hat{q}_{kr}\}$.

Output q_{kr}^* for all $r \in R$ and the optimal sailing times for all legs in plan k.

for $i = 1$ to I **do**

  if $\sum_{r\in R_i} \hat{q}_{kr} \leq Q_i$ **then**

    we have $q_{kr}^* = \hat{q}_{kr}$, $\forall r \in R_i$.

  else

    Define $q_{kr}^* = \hat{q}_{kr}$, $\forall r \in R_i$.

    for $index = 1$ to $\sum_{r\in R_i} \hat{q}_{kr} - Q_i$ **do**

      Find out an $r^* \in \arg\min_{r\in R_i} \theta_{kr,q_{kr}^*}$.

      Set $q_{kr^*}^* \leftarrow q_{kr^*}^* - 1$.

    end for

  end if

end for

for $r \in R$ **do**

  Design the optimal sailing time of each leg on route r by Algorithm 1 with the number of used ships q_{kr}^*.

end for

EC.8. Proof of Proposition 4

Proof. We analyze special case 1, i.e., a case of only one route r including only one VSRIP port p that is called only once, with the increase of the number of deployed ships on the route as follows. Without considering the VSRIP (when 0 nm VSRZ is chosen), we present a minimum total cost function of the route $f_r(\tilde{q}_r)$, including fuel cost and operating cost, when $\tilde{q}_r$ ships are deployed, and the optimal sailing speed on route r should be constant in the function by Lemma 1.

$$f_r(\tilde{q}_r) := w \cdot d_r \cdot \alpha_{i_r}(\frac{d_r}{T\tilde{q}_r - T_r})^{\beta_{i_r}} + \bar{c}_{i_r}\tilde{q}_r, \tag{EC.16}$$

where d_r is the total sailing distance of route r, and $\tilde{q}_r$ is a real number that is no less than the minimum integer number of deployed ships $q_r^{\min}$ on route r calculated by letting ships sail at the maximum physical speed.

If the VSRIP port p includes only one non-zero VSRZ 1 with a radius of d_{p1} and dockage refund $\tilde{c}_{ip1}$ and the VSRZ is chosen, the optimal sailing speeds within and outside the VSRZ on the route are designed by Algorithm 1, and the minimum total cost function of the route $f_r^1(\tilde{q}_r)$ with $\tilde{q}_r$ ships consisting of fuel cost and operating cost minus dockage refund is proposed.

$$f_r^1(\tilde{q}_r)=\begin{cases} w\cdot(d_r-2d_{p1})\cdot\alpha_{i_r}\left(\dfrac{d_r-2d_{p1}}{T\tilde{q}_r-T_r-\frac{2d_{p1}}{V_p^U}}\right)^{\beta_{i_r}}+w\cdot 2d_{p1}\cdot\alpha_{i_r}(V_p^U)^{\beta_{i_r}}+\overline{c}_{i_r}\tilde{q}_r-\tilde{c}_{ip1} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\text{if } (q_r^{\min})^1\le\tilde{q}_r<\frac{d_r+T_r\cdot V_p^U}{T\cdot V_p^U} \\ w\cdot d_r\cdot\alpha_{i_r}\left(\frac{d_r}{T\tilde{q}_r-T_r}\right)^{\beta_{i_r}}+\overline{c}_{i_r}\tilde{q}_r-\tilde{c}_{ip1} \qquad\qquad \text{if } \tilde{q}_r\ge\frac{d_r+T_r\cdot V_p^U}{T\cdot V_p^U}\end{cases}, \quad \text{(EC.17)}$$

where V_p^U is the upper speed limit for each VSRZ of port p, and $(q_r^{\min})^1$ is the minimum integer number of deployed ships on route r calculated by letting ships sail at the speed limit within VSRZ 1 and at the maximum physical speed outside the VSRZ ($(q_r^{\min})^1\ge q_r^{\min}$). The increased cost after following the rules in VSRZ 1 is calculated as:

$$Gap_r^1(\tilde{q}_r)=\begin{cases} w\cdot(d_r-2d_{p1})\cdot\alpha_{i_r}\left(\dfrac{d_r-2d_{p1}}{T\tilde{q}_r-T_r-\frac{2d_{p1}}{V_p^U}}\right)^{\beta_{i_r}}+w\cdot 2d_{p1}\cdot\alpha_{i_r}(V_p^U)^{\beta_{i_r}} \\ -w\cdot d_r\cdot\alpha_{i_r}\left(\frac{d_r}{T\tilde{q}_r-T_r}\right)^{\beta_{i_r}}-\tilde{c}_{ip1} \qquad\qquad \text{if } (q_r^{\min})^1\le\tilde{q}_r<\frac{d_r+T_r\cdot V_p^U}{T\cdot V_p^U} \\ -\tilde{c}_{ip1} \qquad\qquad\qquad\qquad\qquad\qquad \text{if } \tilde{q}_r\ge\frac{d_r+T_r\cdot V_p^U}{T\cdot V_p^U}\end{cases}, \quad \text{(EC.18)}$$

whose first order derivative is

$$\frac{dGap_r^1(\tilde{q}_r)}{d\tilde{q}_r}=\begin{cases} -w\cdot\alpha_{i_r}\cdot\beta_{i_r}\cdot\left(\dfrac{d_r-2d_{p1}}{T\tilde{q}_r-T_r-\frac{2d_{p1}}{V_p^U}}\right)^{\beta_{i_r}+1}\cdot T+w\cdot\alpha_{i_r}\cdot\beta_{i_r}\cdot\left(\dfrac{d_r}{T\tilde{q}_r-T_r}\right)^{\beta_{i_r}+1}\cdot T \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{if } (q_r^{\min})^1\le\tilde{q}_r<\frac{d_r+T_r\cdot V_p^U}{T\cdot V_p^U} \\ 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{if } \tilde{q}_r\ge\frac{d_r+T_r\cdot V_p^U}{T\cdot V_p^U}\end{cases}. \quad \text{(EC.19)}$$

The function $Gap_r^1(\tilde{q}_r)$ is continuous since its first and second pieces are equal when $\tilde{q}_r=\frac{d_r+T_r\cdot V_p^U}{T\cdot V_p^U}$, we have $Gap_r^1(\tilde{q}_r)<0$ when $\tilde{q}_r\ge\frac{d_r+T_r\cdot V_p^U}{T\cdot V_p^U}$ meaning that ships will participate in the VSRIP in this interval since the total cost when VSRZ 1 is chosen is lower than that when 0 nm VSRZ is chosen, and $dGap_r^1(\tilde{q}_r)/d\tilde{q}_r<0$ when $(q_r^{\min})^1\le\tilde{q}_r<\frac{d_r+T_r\cdot V_p^U}{T\cdot V_p^U}$. Therefore, there exists a threshold, denoted by $\tilde{q}_{rp}^{\text{comp1}}$, such that ships on the route will participate in the VSRIP if and only if $\tilde{q}_r\ge\tilde{q}_{rp}^{\text{comp1}}$. If $Gap_r^1((q_r^{\min})^1)\le 0$, $(q_r^{\min})^1$ is the threshold $\tilde{q}_{rp}^{\text{comp1}}$; otherwise, there exists a threshold $\tilde{q}_{rp}^{\text{comp1}}$ between $(q_r^{\min})^1$ and $\frac{d_r+T_r\cdot V_p^U}{T\cdot V_p^U}$ where $Gap_r^1(\tilde{q}_{rp}^{\text{comp1}})=0$.

If the VSRIP port p has two non-zero VSRZs, i.e., VSRZs 1 and 2, with a radius of d_{p1} and dockage refund $\tilde{c}_{ip1}$ for VSRZ 1 and d_{p2} ($d_{p1}<d_{p2}$) and $\tilde{c}_{ip2}$ ($\tilde{c}_{ip1}<\tilde{c}_{ip2}$) for VSRZ 2 and $\tilde{q}_r$ ships are deployed on route r, the minimum total cost function is $f_r^1(\tilde{q}_r)$ when VSRZ 1 is chosen, and similarly, we propose the minimum total cost function $f_r^2(\tilde{q}_r)$ when VSRZ 2 is chosen:

$$f_r^2(\tilde{q}_r) = \begin{cases} w\cdot(d_r - 2d_{p2})\cdot\alpha_{i_r}\left(\dfrac{d_r - 2d_{p2}}{T\tilde{q}_r - T_r - \frac{2d_{p2}}{V_p^U}}\right)^{\beta_{i_r}} + w\cdot 2d_{p2}\cdot\alpha_{i_r}(V_p^U)^{\beta_{i_r}} + \overline{c}_{i_r}\tilde{q}_r - \tilde{c}_{ip2} & \\ & \text{if } (q_r^{\min})^2 \le \tilde{q}_r < \frac{d_r + T_r\cdot V_p^U}{T\cdot V_p^U} \\ w\cdot d_r\cdot\alpha_{i_r}\left(\frac{d_r}{T\tilde{q}_r - T_r}\right)^{\beta_{i_r}} + \overline{c}_{i_r}\tilde{q}_r - \tilde{c}_{ip2} & \text{if } \tilde{q}_r \ge \frac{d_r + T_r\cdot V_p^U}{T\cdot V_p^U} \end{cases}, \quad \text{(EC.20)}$$

where $(q_r^{\min})^2$ is the minimum integer number of deployed ships on route r calculated by letting ships sail at the speed limit within VSRZ 2 and at the maximum physical speed outside the VSRZ $((q_r^{\min})^2 \ge (q_r^{\min})^1)$. Compared with $f_r(\tilde{q}_r)$, the increased cost of $f_r^2(\tilde{q}_r)$ is

$$Gap_r^2(\tilde{q}_r) = \begin{cases} w\cdot(d_r - 2d_{p2})\cdot\alpha_{i_r}\left(\dfrac{d_r - 2d_{p2}}{T\tilde{q}_r - T_r - \frac{2d_{p2}}{V_p^U}}\right)^{\beta_{i_r}} + w\cdot 2d_{p2}\cdot\alpha_{i_r}(V_p^U)^{\beta_{i_r}} & \\ -w\cdot d_r\cdot\alpha_{i_r}\left(\frac{d_r}{T\tilde{q}_r - T_r}\right)^{\beta_{i_r}} - \tilde{c}_{ip2} & \text{if } (q_r^{\min})^2 \le \tilde{q}_r < \frac{d_r + T_r\cdot V_p^U}{T\cdot V_p^U} \\ -\tilde{c}_{ip2} & \text{if } \tilde{q}_r \ge \frac{d_r + T_r\cdot V_p^U}{T\cdot V_p^U} \end{cases} \quad \text{(EC.21)}$$

with the first order derivative

$$\frac{dGap_r^2(\tilde{q}_r)}{d\tilde{q}_r} = \begin{cases} -w\cdot\alpha_{i_r}\cdot\beta_{i_r}\cdot\left(\dfrac{d_r - 2d_{p2}}{T\tilde{q}_r - T_r - \frac{2d_{p2}}{V_p^U}}\right)^{\beta_{i_r}+1}\cdot T + w\cdot\alpha_{i_r}\cdot\beta_{i_r}\cdot\left(\dfrac{d_r}{T\tilde{q}_r - T_r}\right)^{\beta_{i_r}+1}\cdot T & \\ & \text{if } (q_r^{\min})^2 \le \tilde{q}_r < \frac{d_r + T_r\cdot V_p^U}{T\cdot V_p^U} \\ 0 & \text{if } \tilde{q}_r \ge \frac{d_r + T_r\cdot V_p^U}{T\cdot V_p^U} \end{cases}. \quad \text{(EC.22)}$$

$Gap_r^1(\tilde{q}_r)$ and $Gap_r^2(\tilde{q}_r)$ are continuous in their domains and decrease with $\tilde{q}_r$ when $\tilde{q}_r < \frac{d_r + T_r\cdot V_p^U}{T\cdot V_p^U}$, and $Gap_r^1(\tilde{q}_r) < 0$ and $Gap_r^2(\tilde{q}_r) < 0$ when $\tilde{q}_r \ge \frac{d_r + T_r\cdot V_p^U}{T\cdot V_p^U}$. Therefore, we can find a threshold, denoted by $\tilde{q}_{rp}^{\text{comp1}}$, such that ships will participate in the program if and only if $\tilde{q}_r \ge \tilde{q}_{rp}^{\text{comp1}}$. If $Gap_r^1((q_r^{\min})^1) \le 0$, $(q_r^{\min})^1$ is the threshold $\tilde{q}_{rp}^{\text{comp1}}$; if $Gap_r^2((q_r^{\min})^2) \le 0$ and $Gap_r^1((q_r^{\min})^2) \ge 0$, $(q_r^{\min})^2$ is the threshold $\tilde{q}_{rp}^{\text{comp1}}$; otherwise, we could obtain a threshold $\tilde{q}_{rp}^{\text{comp1}}$ during the interval from $(q_r^{\min})^1$ to $(q_r^{\min})^2$ where $Gap_r^1(\tilde{q}_{rp}^{\text{comp1}}) = 0$ or during the interval from $(q_r^{\min})^2$ to $\frac{d_r + T_r\cdot V_p^U}{T\cdot V_p^U}$ where $Gap_r^1(\tilde{q}_{rp}^{\text{comp1}}) = 0$ and $Gap_r^2(\tilde{q}_{rp}^{\text{comp1}}) \ge 0$ or $Gap_r^1(\tilde{q}_{rp}^{\text{comp1}}) \ge 0$ and $Gap_r^2(\tilde{q}_{rp}^{\text{comp1}}) = 0$. Due to $Gap_r^2(\tilde{q}_r) < Gap_r^1(\tilde{q}_r) < 0$ when $\tilde{q}_r \ge \frac{d_r + T_r\cdot V_p^U}{T\cdot V_p^U}$, VSRZ 2 will always be chosen in this interval. We also have $dGap_r^2(\tilde{q}_r)/d\tilde{q}_r < dGap_r^1(\tilde{q}_r)/d\tilde{q}_r < 0$ when $(q_r^{\min})^2 \le \tilde{q}_r < \frac{d_r + T_r\cdot V_p^U}{T\cdot V_p^U}$. Hence, there exists another threshold, denoted by $\tilde{q}_{rp}^{\text{comp2}}$, such that VSRZ 2 will be chosen if and only if $\tilde{q}_r \ge \tilde{q}_{rp}^{\text{comp2}}$. If $\tilde{q}_{rp}^{\text{comp1}} \in [(q_r^{\min})^1, (q_r^{\min})^2)$ or if $\tilde{q}_{rp}^{\text{comp1}} \ge (q_r^{\min})^2$ with $Gap_r^1(\tilde{q}_{rp}^{\text{comp1}}) < Gap_r^2(\tilde{q}_{rp}^{\text{comp1}})$, the threshold $\tilde{q}_{rp}^{\text{comp2}}$ lies in the interval of $(\tilde{q}_{rp}^{\text{comp1}}, \frac{d_r + T_r\cdot V_p^U}{T\cdot V_p^U})$, i.e., VSRZ 1 and VSRZ 2 will be chosen when $\tilde{q}_r \in [\tilde{q}_{rp}^{\text{comp1}}, \tilde{q}_{rp}^{\text{comp2}})$ and when

$\tilde{q}_r \geq \tilde{q}_{rp}^{\text{comp2}}$, respectively; otherwise, we have $\tilde{q}_{rp}^{\text{comp1}} = \tilde{q}_{rp}^{\text{comp2}}$, and VSRZ 2 will always be the best choice when $\tilde{q}_r \geq \tilde{q}_{rp}^{\text{comp1}}$.

It makes sense that the findings above on the real number $\tilde{q}_r$ is also applicable to the integer number q_r. When there is only one non-zero VSRZ at the VSRIP port p, there exists a threshold of the integer number of deployed ships $q_{rp}^{\text{comp1}} = \lceil \tilde{q}_{rp}^{\text{comp1}} \rceil$ ($\lceil z \rceil$ is the smallest integer greater than or equal to z) such that the rules of the program will be complied with if and only if $q_r \geq q_{rp}^{\text{comp1}}$; when there are two non-zero VSRZs, the chosen VSRZ will change from VSRZ 1 to VSRZ 2 at a threshold $q_{rp}^{\text{comp2}} = \lceil \tilde{q}_{rp}^{\text{comp2}} \rceil$ if $q_{rp}^{\text{comp1}} < q_{rp}^{\text{comp2}}$, and otherwise VSRZ 2 will always be chosen when $q_r \geq q_{rp}^{\text{comp1}}$. $\square$

EC.9. Proof of Proposition 5

Proof. We study special case 2, a case with only one route r including $|P^S|$ ($|P^S| \geq 2$) VSRIP ports, with the increase of the number of deployed ships on the route, where each VSRIP port has only one non-zero VSRZ 1 and is called only once, all VSRZs have the same radius d_{11} and speed limit V_1^U, and the dockage refunds at the $|P^S|$ VSRIP ports are all different. Denote by $p^{(1)}, ..., p^{(|P^S|)}$ the VSRIP ports in the decreasing order of dockage refund.

(i) The increased cost of route r with q_r ships when the rules of VSRIPs at n ports are complied with is calculated as the minimum total cost of participating in these VSRIPs minus that of not participating in any VSRIP (0 nm VSRZs are chosen in all VSRIPs):

$$Gap_{r,n,a}(q_r) = \begin{cases} w \cdot (d_r - 2nd_{11}) \cdot \alpha_{i_r} \left(\dfrac{d_r - 2nd_{11}}{Tq_r - T_r - \frac{2nd_{11}}{V_1^U}} \right)^{\beta_{i_r}} + w \cdot 2nd_{11} \cdot \alpha_{i_r} (V_1^U)^{\beta_{i_r}} & \\ -w \cdot d_r \cdot \alpha_{i_r} (\frac{d_r}{Tq_r - T_r})^{\beta_{i_r}} - \sum_{p \in P_{n,a}} \tilde{c}_{ip1} & \text{if } q_{rn}^{\min} \leq q_r < \left\lceil \frac{d_r + T_r \cdot V_1^U}{T \cdot V_1^U} \right\rceil, q_r \in \mathbb{N} \\ -\sum_{p \in P_{n,a}} \tilde{c}_{ip1} & \text{if } q_r \geq \left\lceil \frac{d_r + T_r \cdot V_1^U}{T \cdot V_1^U} \right\rceil, q_r \in \mathbb{N} \end{cases}, \quad \text{(EC.23)}$$

where $P_{n,a}$ is a set of n VSRIP ports chosen from P^S in option a among $\binom{|P^S|}{n}$ choices, and $q_{rn}^{\min}$ is the minimum integer number of deployed ships on route r calculated by letting ships sail at the speed limit within the chosen n non-zero VSRZs and at the maximum physical speed outside these VSRZs. It is evident that ships will participate in all VSRIPs when $q_r \geq \left\lceil \frac{d_r + T_r \cdot V_1^U}{T \cdot V_1^U} \right\rceil$. Next, we will analyze the compliance of VSRIPs when $q_r < \left\lceil \frac{d_r + T_r \cdot V_1^U}{T \cdot V_1^U} \right\rceil$. In the studied case, $Gap_{r,n,a}(q_r)$ can be regarded as the gap between the minimum total cost of participating in a VSRIP with a nd_{11} VSRZ, V_1^U speed limit and $\sum_{p \in P_{n,a}} \tilde{c}_{ip1}$ dockage refund and that of not participating in any VSRIP. Combined with the findings in EC.8, we derive that ships will participate in more VSRIPs with the increase of the number of deployed ships, and if $Gap_{r,|P^S|,1}(q_{r,|P^S|}^{\min})$ is negative and the minimum one among $Gap_{r,n,a}(q_r)$ for all $n = 1, ..., |P^S|$ and $a = 1, ..., \binom{|P^S|}{n}$, the rules of the VSRIPs at all ports $p \in P^S$ will be complied with when $q_r \geq q_{r,|P^S|}^{\min}$. Given the number of chosen VSRIPs

n, the only difference of $Gap_{r,n,a}(q_r)$ among $\binom{|P^S|}{n}$ choices is the total dockage refund provided by the chosen VSRIP ports, and thus ships on the route will participate in a VSRIP with the higher dockage refund first. In conclusion, there exist $|P^S|$ thresholds, denoted by $q^{\text{comp}}_{rp^{(1)}}, ..., q^{\text{comp}}_{rp^{(|P^S|)}}$, $q^{\text{comp}}_{rp^{(1)}} \leq q^{\text{comp}}_{rp^{(2)}} \leq ... \leq q^{\text{comp}}_{rp^{(|P^S|)}}$, such that the rules of the VSRIP at port $p^{(n)}, n = 1, ..., |P^S|$ will be complied with if and only if $q_r \geq q^{\text{comp}}_{rp^{(n)}}$.

(ii) Two situations in special case 2 (case 2.1 and case 2.2) with the only difference on dockage refunds of the $|P^S|$ VSRIPs are considered. For VSRIP port $p \in P^S$ with the same dockage refund $\tilde{c}_{ip1}$ in the two cases, we have $p = p^{(1)}$ in case 2.1 and $p = p^{(n)}, n \geq 2$ in case 2.2. It is shown in Eq. (EC.23) that when $q_r \geq \left\lceil \frac{d_r + T_r \cdot V_1^U}{T \cdot V_1^U} \right\rceil$, the rules of the VSRIPs at all ports will be complied with. Thus, we only focus on the case when $q_r < \left\lceil \frac{d_r + T_r \cdot V_1^U}{T \cdot V_1^U} \right\rceil$.

In case 2.1, the program at port p is the first chosen VSRIP whose rules will be complied with by the finding in (i) and it will be chosen only when $\hat{Gap}_{r,1}(q_r) \leq 0$, where $\hat{Gap}_{r,1}(q_r)$ is the minimum one among $Gap_{r,1,a}(q_r)$ for all $a = 1, ..., |P^S|$.

$$\begin{aligned}\hat{Gap}_{r,1}(q_r) = {} & w \cdot (d_r - 2d_{11}) \cdot \alpha_{i_r} \left(\frac{d_r - 2d_{11}}{Tq_r - T_r - \frac{2d_{11}}{V_1^U}} \right)^{\beta_{i_r}} + w \cdot 2d_{11} \cdot \alpha_{i_r} (V_1^U)^{\beta_{i_r}} \\ & - w \cdot d_r \cdot \alpha_{i_r} \left(\frac{d_r}{Tq_r - T_r} \right)^{\beta_{i_r}} - \tilde{c}_{ip1}. \end{aligned} \tag{EC.24}$$

In case 2.2, ships will participate in the VSRIP at port p only when the following inequalities are satisfied: $\hat{Gap}'_{r,n}(q_r) \leq 0$ and $\hat{Gap}'_{r,n}(q_r) - \hat{Gap}'_{r,\xi}(q_r) \leq 0$ for all $\xi = 1, ..., n-1$, where $\hat{Gap}'_{r,\xi}(q_r), \xi = 1, ..., n$ is the minimum one among $Gap_{r,\xi,a}(q_r)$ for all $a = 1, ..., \binom{|P^S|}{\xi}$. Comparing $\hat{Gap}_{r,1}(q_r)$ with $\hat{Gap}'_{r,n}(q_r) - \hat{Gap}'_{r,n-1}(q_r)$ by developing a new function $GAP_r(q_r) = (\hat{Gap}'_{r,n}(q_r) - \hat{Gap}'_{r,n-1}(q_r)) - \hat{Gap}_{r,1}(q_r)$, we have

$$\begin{aligned}GAP_r(q_r) = {} & w \cdot (d_r - 2nd_{11}) \cdot \alpha_{i_r} \left(\frac{d_r - 2nd_{11}}{Tq_r - T_r - \frac{2nd_{11}}{V_1^U}} \right)^{\beta_{i_r}} - w \cdot [d_r - 2(n-1)d_{11}] \cdot \alpha_{i_r} \left[\frac{d_r - 2(n-1)d_{11}}{Tq_r - T_r - \frac{2(n-1)d_{11}}{V_1^U}} \right]^{\beta_{i_r}} \\ & - w \cdot (d_r - 2d_{11}) \cdot \alpha_{i_r} \left(\frac{d_r - 2d_{11}}{Tq_r - T_r - \frac{2d_{11}}{V_1^U}} \right)^{\beta_{i_r}} + w \cdot d_r \cdot \alpha_{i_r} \left(\frac{d_r}{Tq_r - T_r} \right)^{\beta_{i_r}}. \end{aligned} \tag{EC.25}$$

To analyze the terms in $GAP_r(q_r)$, we define a function $h(\tilde{d}_{11})$, where $\tilde{d}_{11}$ is a real number between 0 and d_r.

$$h(\tilde{d}_{11}) = w \cdot (d_r - \tilde{d}_{11}) \cdot \alpha_{i_r} \left(\frac{d_r - \tilde{d}_{11}}{Tq_r - T_r - \frac{\tilde{d}_{11}}{V_1^U}} \right)^{\beta_{i_r}}, \tag{EC.26}$$

whose first and second order derivatives require

$$\frac{dh(\tilde{d}_{11})}{d\tilde{d}_{11}} = w \cdot \alpha_{i_r} (d_r - \tilde{d}_{11})^{\beta_{i_r}} (Tq_r - T_r - \frac{\tilde{d}_{11}}{V_1^U})^{-\beta_{i_r}} \left[\beta_{i_r} (d_r - \tilde{d}_{11}) (Tq_r - T_r - \frac{\tilde{d}_{11}}{V_1^U})^{-1} \frac{1}{V_1^U} - (\beta_{i_r} + 1) \right] \tag{EC.27}$$

$$\begin{aligned}&\frac{d^2h(\tilde{d}_{11})}{d(\tilde{d}_{11})^2}\\&=w\cdot\alpha_{i_r}\beta_{i_r}(\beta_{i_r}+1)(d_r-\tilde{d}_{11})^{\beta_{i_r}-1}(Tq_r-T_r-\frac{\tilde{d}_{11}}{V_1^U})^{-\beta_{i_r}}\left[(d_r-\tilde{d}_{11})(Tq_r-T_r-\frac{\tilde{d}_{11}}{V_1^U})^{-1}\frac{1}{V_1^U}-1\right]^2. \quad \text{(EC.28)}\end{aligned}$$

We infer that $(d_r-\tilde{d}_{11})(Tq_r-T_r-\frac{\tilde{d}_{11}}{V_1^U})^{-1}\frac{1}{V_1^U}-1>0$ when $q_r<\left\lceil\frac{d_r+T_r\cdot V_1^U}{T\cdot V_1^U}\right\rceil$, and thus $d^2h(\tilde{d}_{11})/d(\tilde{d}_{11})^2>0$, which indicates that $h(\tilde{d}_{11})$ is a strictly convex function.

According to the property of $h(\tilde{d}_{11})$, we have $w\cdot(d_r-2nd_{11})\cdot\alpha_{i_r}\left(\frac{d_r-2nd_{11}}{Tq_r-T_r-\frac{2nd_{11}}{V_1^U}}\right)^{\beta_{i_r}}-w\cdot[d_r-2(n-1)d_{11}]\cdot\alpha_{i_r}\left[\frac{d_r-2(n-1)d_{11}}{Tq_r-T_r-\frac{2(n-1)d_{11}}{V_1^U}}\right]^{\beta_{i_r}}>w\cdot(d_r-2d_{11})\cdot\alpha_{i_r}\left(\frac{d_r-2d_{11}}{Tq_r-T_r-\frac{2d_{11}}{V_1^U}}\right)^{\beta_{i_r}}-w\cdot d_r\cdot\alpha_{i_r}\left(\frac{d_r}{Tq_r-T_r}\right)^{\beta_{i_r}}$, i.e., $GAP_r(q_r)>0$. We can find two thresholds of port p in the two cases, denoted by $q_{rp^{(1)}}^{\text{comp}\prime}$ when $p=p^{(1)}$ and $q_{rp^{(n)}}^{\text{comp}\prime}$ when $p=p^{(n)}$, such that ships will participate in the VSRIP at port p in cases 2.1 and 2.2 if and only if $q_r\geq q_{rp^{(1)}}^{\text{comp}\prime}$ and $q_r\geq q_{rp^{(n)}}^{\text{comp}\prime}$, respectively, and we have $q_{rp^{(1)}}^{\text{comp}\prime}\leq q_{rp^{(n)}}^{\text{comp}\prime}$ since $\hat{Gap}_{r,1}(q_r)<\hat{Gap}'_{r,n}(q_r)-\hat{Gap}'_{r,n-1}(q_r)$ when $q_r<\left\lceil\frac{d_r+T_r\cdot V_1^U}{T\cdot V_1^U}\right\rceil$. □

EC.10. Proof of Proposition 6

Proof. The proposition can be proved by induction.

(i) When $B_{rl}=1$, there exist two cases: If $\epsilon_{rl}^0=\epsilon_{rl}$, our algorithm can find the unique approximation line, which cannot be moved or deleted; if $\epsilon_{rl}^0<\epsilon_{rl}$, this line also cannot be deleted, but it can be substituted by other feasible approximation lines.

(ii) Suppose that no approximation line can be reduced when $B_{rl}=b$, $b\geq 1$ in order to guarantee the approximation error no greater than ϵ_{rl} and the lines generated by our algorithm are the unique solution if $\epsilon_{rl}^0=\epsilon_{rl}$.

(iii) When $B_{rl}=b+1$, $b\geq 1$, define the domain of an approximation line as the area within which the approximation error between this line and the function $F_{rl}(t_{rl})$ is less than or equal to ϵ_{rl} and outside which the corresponding approximation error is greater than ϵ_{rl}. The domain of approximation lines 1 to b is denoted by $[\underline{t}_{rl},t'_{rl}]$ and the domain of approximation line $b+1$ is denoted by $[t'_{rl},\bar{t}_{rl}]$. There are two cases by comparing ϵ_{rl}^0 with ϵ_{rl}, i.e., $\epsilon_{rl}^0=\epsilon_{rl}$ (Case one, see Fig. EC.1a) and $\epsilon_{rl}^0<\epsilon_{rl}$ (Case two, see Fig. EC.1b). In Case one, we assume there exists another algorithm which can compute a new solution with no more than $b+1$ approximation lines. According to the assumption in (ii), lines 1 to b cannot be moved. If line $b+1$ remains in the new solution, the new solution is the same as the solution obtained from our algorithm. If line $b+1$ is changed to line $(b+1)'$, the domain of approximation lines 1 to b is recorded as $[\underline{t}_{rl},t''_{rl}]$ and the domain of approximation line $(b+1)'$ is $[t''_{rl},\bar{t}_{rl}]$. The approximation error between $F_{rl}(t_{rl})$ and line $(b+1)'$ at t'_{rl}, i.e., segment f_1f_2 in Fig. EC.1a, will be greater than ϵ_{rl} to ascertain that the error of the two functions at $\bar{t}_{rl}$ is less than or equal to ϵ_{rl}. Therefore the error between the function $F_{rl}(t_{rl})$ and the intersection point of lines b and $(b+1)'$, i.e., segment f_3f_4 in Fig. EC.1a, at t''_{rl} is greater than ϵ_{rl}, which means the new solution is not feasible. Hence we conclude that the proposed solution by our algorithm is unique. In Case two, b lines are necessary and cannot be moved in the domain $[\underline{t}_{rl}, t'_{rl}]$ by the assumption in (ii) and we need one more line to cover the domain $[t'_{rl}, \bar{t}_{rl}]$. As a result, $b+1$ is the smallest number of approximation lines if the approximation error is no greater than the given value ϵ_{rl}.

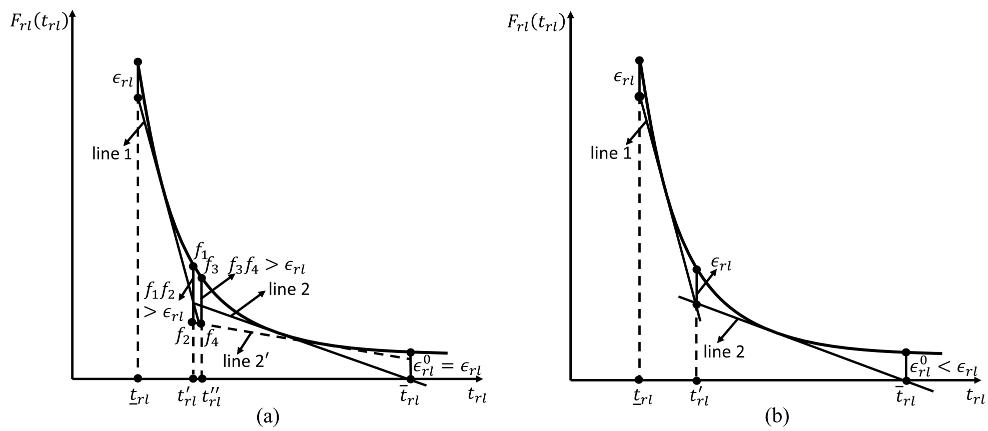

Figure EC.1 Two cases of approximation lines

In summary, the number of lines B_{rl} is the smallest for a given approximation error ϵ_{rl} and our algorithm can find out the unique solution when $\epsilon^0_{rl} = \epsilon_{rl}$. □

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